

Transformations and OSGM02 User guide



Preface

This grid transformation software and user guide has been commissioned by a consortium comprising Ordnance Survey Great Britain, Ordnance Survey Ireland (OSi), and Ordnance Survey of Northern Ireland (OSNI). These organisations are responsible for the official, definitive topographic mapping of their respective countries.

This user guide contains all the information you need to make effective use of the Ordnance Survey Grid Transformations in the UK and Ireland, and the Ordnance Survey Geoid model (OSGM02™). Within Great Britain coordinates are transformed using the Ordnance Survey National Grid Transformation (OSTN02™). Within the Republic of Ireland and Northern Ireland, the OSi/OSNI Polynomial Transformation is used. OSGM02 is used to transform heights throughout the UK and Ireland.

Users wishing to incorporate the pre-prepared .dll should refer to the Grid InQuest 6.0 user guide.

The intention of this guide is to help you understand the information contained in the data, as well as providing detailed technical information and the data format specification; however it is assumed that users have some appreciation of coordinate systems and datums. This user guide has been checked and validated before issue and every endeavour made to ensure that the contents are accurate. If you find an error, omission, or otherwise wish to make a suggestion as to how this user guide can be improved, please contact us at the address shown under Contact details.

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Chapter 1 Introduction

Coordinate transformations and the Geoid model

All Ordnance Survey mapping relates to a coordinate reference system. In Great Britain Ordnance Survey coordinates relate to OSGB36® (the National Grid); within Northern Ireland and the Republic of Ireland the coordinate reference system is the Irish Grid. These reference systems are traditionally realised on the earth's surface by monumented triangulation stations. The users of mapping products, in both the public and private sectors, have invested in geographical information systems (GIS) and asset management systems based on these grid systems, which have been accepted as de facto national standards.

The National Grid and the Irish Grid are capable of supporting surveying and mapping in UK and Ireland to meet all the requirements of users both now and in the future; however, an increasing number of spatial datasets are available in GPS (Global Positioning System) coordinate systems. When two or more coordinate datasets are to be integrated, it is essential that each relates to the same coordinate reference system, irrespective of accuracy issues.

In order to relate GPS-derived positions to Ordnance Survey's mapping, GPS coordinates need to be converted to Irish Grid or to National Grid, which requires a specialised datum transformation. For this reason the Ordnance Survey of Northern Ireland and Ordnance Survey Ireland have developed a polynomial transformation, which is the standard datum transformation for use throughout Ireland. The Ordnance Survey of Great Britain has developed OSTN02, the standard datum transformation for Great Britain.

Ordnance Survey mapping also includes height information that relates to a regional vertical datum. Height information in Great Britain refers to Ordnance Datum Newlyn (ODN), which is established from mean sea level. Although ODN is the national height datum used across mainland Great Britain there are a number of additional datums that are used on the surrounding islands, for example: Lerwick on the Shetland Islands; Stornoway on the Outer Hebrides; St Kilda; Douglas02 on the Isle of Man and St Marys on the Scilly Isles. The Ordnance Survey of Northern Ireland relates heights within Northern Ireland to Belfast Lough datum, and Ordnance Survey Ireland relates heights within the Republic of Ireland to the Malin Head datum.

Orthometric heights in these systems have in the past been realised via a network of Ordnance Survey bench marks (BMs). These traditional levelling networks cover the whole of Great Britain, Northern Ireland and the Republic of Ireland. However, heights from precise GPS surveying are relative to a reference ellipsoid that approximates to the shape of the earth but does not coincide with mean sea level. To enable GPS to be used to determine orthometric heights, the Ordnance Surveys have jointly developed a model to establish the precise relationship between the two vertical reference surfaces. The resulting Ordnance Survey Geoid model (OSGM02) incorporates all the above vertical datums.

OSTN02

The Ordnance Survey of Great Britain has developed the horizontal transformation OSTN02. This transformation consists of a 700 km by 1 250 km grid of translation vectors at 1 km resolution. This provides a fit between the GPS coordinate system ETRS89 (European Terrestrial Reference System 1989) and the OSGB36 National Grid. OSTN02 is in agreement with major triangulation stations at the level of 0.1 m root mean square error (RMSE).

OSTN02 has been developed from the national primary, secondary and tertiary triangulation station network. It contains over 3 200 points directly observed by GPS and more than 1 000 from the original retriangulation observations adjusted on the ETRS89 datum.

Within Great Britain OSTN02 (the Ordnance Survey National Grid Transformation), in conjunction with the ETRS89 positions of the active GPS network stations, is now the official definition of OSGB36 National Grid coordinate system. This means that using OSTN02 with the National GPS Network, surveyors using GPS have no need to occupy triangulation stations in order to relate GPS coordinates to National Grid coordinates.

OSi/OSNI polynomial transformation

Ordnance Survey Ireland and Ordnance Survey of Northern Ireland recommend the OSi/OSNI polynomial transformation for all horizontal transformations in the Republic of Ireland and Northern Ireland. This transformation has been developed in association with the Institute of Engineering Surveying and Space Geodesy, University of Nottingham.

The transformation is based on 183 points evenly distributed throughout Ireland and Northern Ireland. The precise ETRS89 and Irish Grid coordinates of these points are determined by GPS and terrestrial survey methods, and a one-dimensional 3rd order polynomial individually fitted to the latitude and the longitude. The resulting polynomial allows calculation of the coordinate differences at additional points. The polynomial transformation has an accuracy of 0.4 m (95% data).

Ordnance Survey Geoid model: OSGM02

To provide the third dimension of the transformation, the Ordnance Surveys have, with others, developed the Geoid model OSGM02. The model is derived from precise gravity surveys across UK, Ireland, and surrounding waters; additionally, the model includes data from the global geopotential model (EGM96). Alignment to each regional vertical datum is based on precise GPS observations at Ordnance Survey levelling marks. Within Great Britain these include the Ordnance Survey fundamental bench mark network.

The Geoid model consists of a 1 km grid with geoid-ellipsoid separation values covering all of Great Britain, Ireland and Northern Ireland. This model can be used with GPS determined positions to establish height above mean sea level, as defined by the respective vertical datums, to the accuracies shown in the table below. The Ordnance Surveys recommend the use of the Geoid model OSGM02 and the active GPS network to produce orthometric height compatible with Ordnance Survey mapping. The standard error of the main datums are:

OSGM02 region	Standard error (m)
Great Britain	0.02
Republic of Ireland	0.03
Northern Ireland	0.02
Orkney	0.08
Shetland	0.03
Outer Hebrides	0.09
Isle of Man	0.03
St Kilda	0.06

Ordnance Survey Great Britain intend that OSGM02 is the official definition of the relationship between GPS ellipsoid heights and orthometric height in Great Britain. **In the way that GPS and the transformation model OSTN02 define the horizontal coordinate system, precise GPS surveying using the Ordnance Survey Great Britain active GPS Network in conjunction with the Geoid model will become the standard method of determining orthometric height.**

ETRS89 explained

The Ordnance Survey transformations and OSGM02 link the Ordnance Survey coordinate reference systems and vertical datums to the GPS-compatible coordinate system ETRS89. In Europe, ETRS89 is a precise version of the better known WGS84 reference system optimised for use in Europe; however, for most purposes it can be considered equivalent to WGS84.

Specifically, the motion of the European continental plate is not apparent in ETRS89, which allows a fixed relationship to be established between this system and Ordnance Survey mapping coordinate systems.

Additional precise versions of WGS84 are currently in use, notably ITRS; these are not equivalent to ETRS89. The difference between ITRS and ETRS89 is in the order of 0.25 m (in 1999), and growing by 0.025 m per year in UK and Ireland. This effect is only relevant in international scientific applications. For all navigation, mapping, GIS, and engineering applications within the tectonically stable parts of Europe (including UK and Ireland), the term ETRS89 should be taken as synonymous with WGS84.

Benefits

Together, the Ordnance Survey transformations and OSGM02 provide the complete solution to relating GPS (WGS84) datasets to Ordnance Survey mapping in three dimensions. Used with the active GPS network, they allow GPS surveying within the National Grid or the Irish Grid, and to the appropriate vertical datum, without the need to visit any Ordnance Survey traditional control points. These new standards bring the benefits of simplicity and uniformity.

Applications

The Ordnance Survey transformations and OSGM02 are of interest to:

- GPS surveyors who need to relate their survey to the National Grid or the Irish Grid and/or OD orthometric heights – used with the active GPS Network, these products remove the need to visit traditional Ordnance Survey horizontal and vertical control points; and
- GIS, GPS, CAD and navigation system developers who need to integrate GPS (WGS84) datasets with Ordnance Survey mapping – these products provide the complete solution to these users at all Ordnance Survey mapping scales.

Chapter 2 Data overview

Basic principles

Specifications of OSTN02 horizontal transformation

Transformation:	horizontal datum transformation between ETRS89 and OSGB36
Transformation type:	interpolated square grids of easting and northing shifts
Estimation method:	Delaunay triangulation
Grid resolution:	1 km
Grid interpolation:	bilinear
Accuracy:	0.1 m (RMS) with respect to OSGB36 primary, secondary and tertiary triangulation monuments
Extent:	700 km east by 1 250 km north

Specifications of OSi/OSNI polynomial transformation

Transformation:	datum transformation between Irish Grid and ETRS89
Transformation type:	3rd order polynomial
Accuracy:	0.4 m (95% of data)
Extent:	Republic of Ireland and Northern Ireland

Specifications of OSGM02 Geoid model

Transformation:	vertical ETRS89 ellipsoid to orthometric height
Transformation type:	interpolated square grid of geoid heights above ETRS89 ellipsoid
Estimation method:	Spherical Fast Fourier transformation with modified Stokes kernels
Grid resolution:	1 km
Grid interpolation:	bilinear
Accuracy:	Area specific: Mainland GB 2 cm rms, Republic of Ireland 3 cm rms, Northern Ireland 2 cm rms, with respect to Ordnance Survey BMs,

Data structure

OSTN02/OSGM02 within Great Britain – format and layout of the data

Within Great Britain OSTN02 and OSGM02 are released as a combined data file using the same 1 km grid. This grid covers an area 700 km east–west and 1 250 km north–south, the origin being the origin of the projected ETRS89 coordinates (see [appendix B](#)). However, the area to which the models are valid has been *cookie cut* to a 10 km polygon beyond the coastline. This means that any position more than 10 km offshore will have fields 4, 5, 6 and 7 (see below) all set to zero.

It is strongly recommended that the transformation software be written to return an *outside transformation boundary* type error when a point outside this area is encountered.

Each record occupies a separate line with the south-west corner of the grid being the first record in the file. The format of each record is indicated by the following table:

Record no ¹	ETRS89 easting ² (m)	ETRS89 northing ³ (m)	OSTN02 east shift ⁴ (m)	OSTN02 north shift ⁵ (m)	OSGM02 Geoid Ht ⁶ (m)	Geoid datum flag ⁷
1	0	0				
2	1 000	0				
3	2 000	0				
and so on	and so on	and so on				
701	700 000	0				
702	0	1 000				
703	1 000	1 000				
704	2 000	1 000				
and so on	and so on	and so on				
876 948	697 000	1 250 000				
876 949	698 000	1 250 000				
876 950	699 000	1 250 000				
876 951	700 000	1 250 000				

Where:

- 1 the record number is a sequential number starting at 1 for the origin point (0,0) and finishing at 876 951 for the north-east corner (700 000, 1 250 000).
- 2 ETRS89, National Grid projection, grid intersection easting coordinate in metres.
- 3 ETRS89, National Grid projection, grid intersection northing coordinate in metres.
- 4 the shift in eastings, at the intersection, between ETRS89 and OSGB36 National Grid, that is $\text{ETRS89 east} + \text{OSTN02 east shift} = \text{OSGB36 National Grid easting}$.
- 5 the shift in northings, at the intersection, between ETRS89 and OSGB36 National Grid, that is $\text{ETRS89 north} + \text{OSTN02 north shift} = \text{OSGB36 National Grid northing}$.
- 6 the height of the Geoid above the ETRS89 ellipsoid, in metres, at the intersection, that is $\text{ETRS89 height} - \text{OSGM02 Geoid height} = \text{orthometric height above mean sea level}$.
- 7 the Geoid datum flag is a number from 1 to 14, representing the geoid datum to which OSGM02 Geoid height relates, for example, this flag is 1 for Ordnance Datum Newlyn, the orthometric height datum used on the British mainland. See table 1 (page 17), for details of other datum flag references.

OSGM02 within Northern Ireland – format and layout of the data

Within Northern Ireland OSGM02 is released as a single data file using a 1 km grid. This grid covers an area 250 km east–west and 200 km north–south, the origin is offset by (550 000, 800 000) from the origin of the projected ETRS89 coordinates (see [appendix B](#)). As with the data for Great Britain, the area to which the models are valid has been *cookie cut* to a polygon around the coastline and the border with the Republic of Ireland. This means that any position more than 10 km offshore or 2 km over the border with the Republic of Ireland will have fields 4 and 5 (see below) set to zero.

It is strongly recommended that the transformation software be written to return an *outside transformation boundary* type error when a point outside this area is encountered.

Each record occupies a separate line with the south-west corner being the first record in the file. The format of each record is indicated by the following table:

Record no ¹	ETRS89 easting ² (m)	ETRS89 northing ³ (m)	OSGM02 Geoid Ht ⁴ (m)	Geoid datum flag ⁵
1	550 000	800 000		
2	551 000	800 000		
3	552 000	800 000		
and so on	and so on	and so on		
251	800 000	800 000		
252	550 000	801 000		
253	551 000	801 000		
254	552 000	801 000		
255	553 000	801 000		
and so on	and so on	and so on		
50 448	797 000	1 000 000		
50 449	798 000	1 000 000		
50 450	799 000	1 000 000		
50 451	800 000	1 000 000		

Where:

- 1 the record number is a sequential number starting at 1 for the origin point (550 000, 800 000) and finishing at 50451 for the north-east corner (800 000, 1 000 000).
- 2 ETRS89, ITM projection, grid intersection easting coordinate in metres.
- 3 ETRS89, ITM projection, grid intersection northing coordinate in metres.
- 4 the height of the Geoid above the ETRS89 ellipsoid, in metres, at the intersection, that is
 $\text{ETRS89 height} - \text{OSGM02 Geoid height} = \text{orthometric height above mean sea level}$.
- 5 the Geoid datum flag is a number from 1 to 14, representing the geoid datum to which OSGM02 Geoid height relates, for example, this flag is 14 for Belfast, the orthometric height datum used in Northern Ireland. See table 1 (page 17), for details of other datum flag references.

OSGM02 within the Republic of Ireland – format and layout of the data

Within the Republic of Ireland OSGM02 is released as a single data file using a 1 km grid. This grid covers an area 350 km east–west and 500 km north–south, the origin is offset by (400 000, 500 000) from the origin of the projected ETRS89 coordinates (see [appendix B](#)). As with the data for Great Britain and Northern Ireland, the area to which the models are valid has been *cookie cut* to a polygon around the coastline and the border with Northern Ireland. This means that any position more than 10 km offshore or 2 km over the border with Northern Ireland will have fields 4 and 5 (see below) set to zero.

It is strongly recommended that the transformation software be written to return an *outside transformation boundary* type error when a point outside this area is encountered.

Each record occupies a separate line with the south-west corner being the first record in the file. The format of each record is indicated by the following table:

Record no ¹	Easting ² (m)	Northing ³ (m)	OSGM02 Geoid Ht ⁴ (m)	Geoid datum flag ⁵
1	400 000	500 000		
2	401 000	500 000		
3	402 000	500 000		
and so on	and so on	and so on		
351	750 000	500 000		
352	400 000	501 000		
353	401 000	501 000		
354	402 000	501 000		
355	403 000	501 000		
and so on	and so on	and so on		
175 848	747 000	1 000 000		
175 849	748 000	1 000 000		
175 850	749 000	1 000 000		
175 851	750 000	1 000 000		

Where:

- 1 the record number is a sequential number starting at 1 for the origin point (0,0) and finishing at 175 851 for the north-east corner (750 000, 1 000 000).
- 2 ETRS89, ITM projection, grid intersection easting coordinate in metres.
- 3 ETRS89, ITM projection, grid intersection northing coordinate in metres.

- 4 the height of the Geoid above the ETRS89 ellipsoid, in metres, at the intersection, that is $\text{ETRS89 height} - \text{OSGM02 Geoid height} = \text{orthometric height above mean sea level}$.
- 5 the Geoid datum flag is a number from 1 to 14, representing the geoid datum to which OSGM02 Geoid height relates, for example, this flag is 13 for Malin Head, the orthometric height datum used in the Republic of Ireland. See table 1 below for details of other datum flag references.

Table 1

Geoid datum flag	Datum name	Region
0	N/A	Outside model boundary
1	Newlyn	UK mainland
2	St Marys	Scilly Isles
3	Douglas02	Isle of Man
4	Stornoway	Outer Hebrides
5	St Kilda	St Kilda
6	Lerwick	Shetland Isles
7	Newlyn	Orkney Isles
8	Fair Isle	Fair Isle
9	Flannan Isles	Flannan Isles
10	North Rona	North Rona
11	Sule Skerry	Sule Skerry
12	Foula	Foula
13	Malin Head	Republic of Ireland
14	Belfast	Northern Ireland

NOTE: If the datum flag indicates a zero (0) then the coordinate falls more than 10 km offshore or 2 km beyond the Northern Ireland/Republic of Ireland border and is not covered by the model.

Chapter 3 Ordnance Survey transformations and OSGM02 explained

This chapter explains the algorithms that must be coded to implement the Ordnance Survey transformations (OSTN02 and the OSi/OSNI polynomial) and OSGM02.

OSTN02

Transforming ETRS89 coordinates to OSGB36 National Grid: overview

To transform an ETRS89 value to OSGB36, the ETRS89 easting and northing is first obtained using the algorithm, GRS80 ellipsoid parameters and National Grid projection parameters in [appendix B](#). Within the kilometre square where the point falls, a bilinear interpolation is used to obtain the exact transformation value for the point from the values at the four corners of the kilometre square. These values are added to the ETRS89 easting and northing to obtain the OSGB36 values. The inverse transformation (OSGB36 to ETRS89) is accomplished by an iterative procedure.

Calculating which data record to use

To find the record number corresponding to a given ETRS89 **easting** and **northing**, use the following algorithm:

east_index	=	integer_part_of (easting/1 000)
north_index	=	integer_part_of (northing/1 000)
record_number	=	east_index + (north_index x 701) + 1

For example, to find the record for (2 000E, 1 000N):

$$\begin{aligned}\text{east_index} &= \text{integer_part_of} (2\,000/1\,000) \\ &= 2 \\ \text{north_index} &= \text{integer_part_of} (1\,000/1\,000) \\ &= 1 \\ \text{record_number} &= \text{east_index} + (\text{north_index} \times 701) + 1 \\ &= 2 + 1 \times 701 + 1 \\ &= 704\end{aligned}$$

Procedure for transforming ETRS89 to OSGB36 coordinates

To convert an ETRS89 easting and northing (x, y) obtained using [appendix B](#) to a National Grid easting and northing (e, n), the easting and northing shifts from the data file should be added to the x and y coordinates, respectively.

The point to be transformed is unlikely to lie exactly on one of the nodes of the grid, so to calculate the shifts at any other points an interpolation is required.

The first stage in the transformation is to identify in which grid cell the ETRS89 point lies. This simply requires an integer division of the (x, y) coordinates, where x and y are in metres:

$$\begin{aligned}\text{east_index} &= \text{integer_part_of} (x/1\,000) \\ \text{north_index} &= \text{integer_part_of} (y/1\,000)\end{aligned}$$

Having located the correct cell, find the values of the shifts at the four corners of the cell (se_0, se_1, se_2, se_3 for the shifts in eastings, and sn_0, sn_1, sn_2, sn_3 for the shifts in northings) and the offsets of the point x, y from the bottom left corner of the cell (x_0, y_0) – shown in figure 1 below (the sg values in the figure are used to find the vertical shift values from OSGM02 later on).

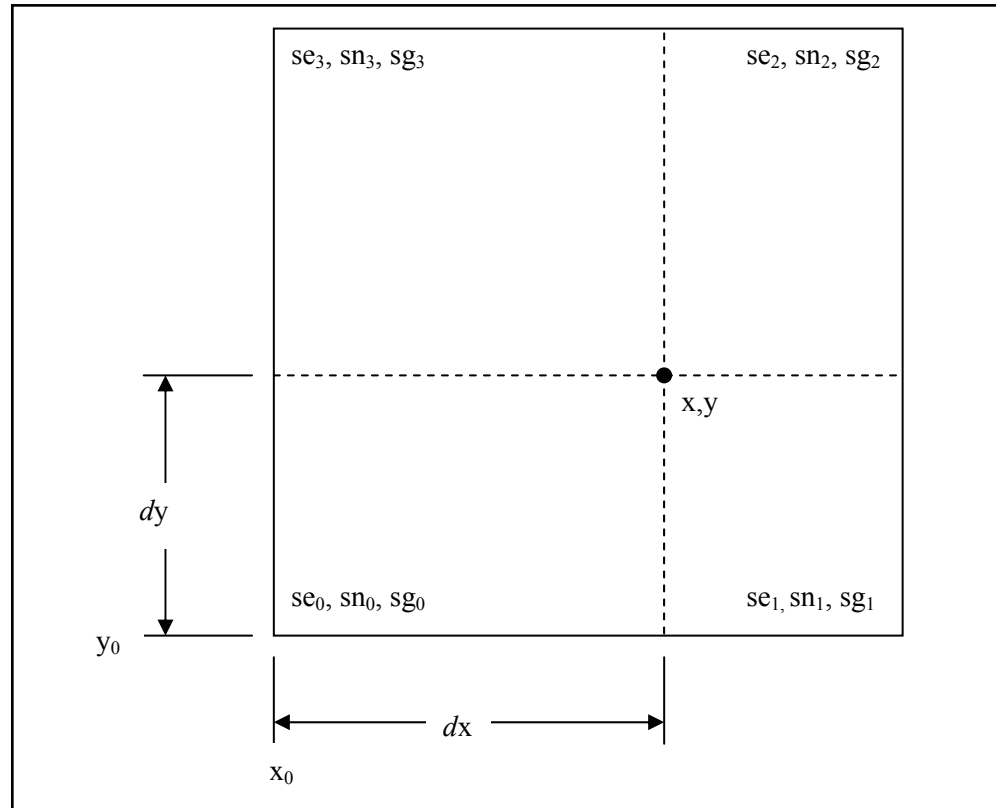


Figure 1: Calculating the OSTN02 se and sn horizontal shifts and the sg vertical shifts for OSGM02.

Shifts for x, y are:

$$se_0 = \text{east_shift}(\text{east_index}, \text{north_index})$$

NOTE: recall that the record number in the data file will be (east_index+(north_index x 701) + 1)

$$se_1 = \text{east_shift}(\text{east_index} + 1, \text{north_index})$$

$$se_2 = \text{east_shift}(\text{east_index} + 1, \text{north_index} + 1)$$

$$se_3 = \text{east_shift}(\text{east_index}, \text{north_index} + 1)$$

$$sn_0 = \text{north_shift}(\text{east_index}, \text{north_index})$$

$$sn_1 = \text{north_shift}(\text{east_index} + 1, \text{north_index})$$

$$sn_2 = \text{north_shift}(\text{east_index} + 1, \text{north_index} + 1)$$

$$sn_3 = \text{north_shift}(\text{east_index}, \text{north_index} + 1)$$

Offsets are:

$$dx = x - x_0$$

$$dy = y - y_0$$

The value of the east shift (se), north shift (sn) at the point x, y is given by the following formulae:

$$t = dx / 1\,000$$

$$u = dy / 1\,000$$

$$se = (1 - t)(1 - u) se_0 + (t)(1 - u) se_1 + (t)(u) se_2 + (1 - t)(u) se_3$$

$$sn = (1 - t)(1 - u) sn_0 + (t)(1 - u) sn_1 + (t)(u) sn_2 + (1 - t)(u) sn_3$$

These shifts must then be added to the point x, y to give the National Grid position (e, n):

$$e = x + se$$

$$n = y + sn$$

Inverse transformation (OSGB36 to ETRS89)

To compute ETRS89 eastings and northings from OSGB36 coordinates, an iterative procedure is required:

Step 1

To start the iteration, compute the ETRS89–OSGB36 easting and northing shifts at the OSGB36 point, using the OSGB36 easting and northing and the method described in procedure for transforming coordinates (page 20).

Subtract these shifts from the OSGB36 coordinates to obtain the first estimate of the ETRS89 easting and northing.

Step 2

Use this estimate of the ETRS89 easting and northing to obtain improved values for the easting and northing shifts, and **subtract** these from the OSGB36 coordinates to obtain improved values of the ETRS89 easting and northing.

Step 3

If the difference between the first shift value and second shift value is more than 0.0001 metres in either easting or northing, repeat step 2 until this is not the case.

Step 4

If ETRS89 latitude and longitude coordinates are required, obtain these from the ETRS89 easting and northing by the procedure described in [appendix C](#).

The OSi/OSNI polynomial transformation

To some extent distortions within traditional triangulation networks are inevitable. Within the triangulation network of the Republic of Ireland and Northern Ireland these distortions are not generally significant; however, regional distortions do occur. A 3rd order polynomial transformation has been developed to model these distortions.

A polynomial expression was fitted to the coordinate differences of a number of points in the different coordinate reference systems. This is a one-dimensional fitting method that is applied to the geographical coordinate, requiring independent parameters to be computed for both latitude and longitude.

In general, the polynomial model can be expressed as:

$$\begin{aligned}\Delta\phi &= \sum\sum A_{ij}(\phi - \phi_m)^i(\lambda - \lambda_m)^j \\ \Delta\lambda &= \sum\sum B_{ij}(\phi - \phi_m)^i(\lambda - \lambda_m)^j\end{aligned}$$

The fully expanded forms of the 3rd order polynomial are as follows:

$$\Delta\phi = [A_{00} + A_{10}U + A_{01}V + A_{11}UV + A_{20}U^2 + A_{02}V^2 + A_{21}U^2V + A_{12}UV^2 + A_{22}U^2V^2 + A_{30}U^3 + A_{03}V^3 + A_{31}U^3V + A_{13}UV^3 + A_{32}U^3V^2 + A_{23}U^2V^3 + A_{33}U^3V^3] / 3600$$

$$\Delta\lambda = [B_{00} + B_{10}U + B_{01}V + B_{11}UV + B_{20}U^2 + B_{02}V^2 + B_{21}U^2V + B_{12}UV^2 + B_{22}U^2V^2 + A_{30}U^3 + B_{03}V^3 + B_{31}U^3V + B_{13}UV^3 + B_{32}U^3V^2 + B_{23}U^2V^3 + B_{33}U^3V^3] / 3600$$

Where A_{ij} and B_{ij} are the computed parameters, and U and V are the normalised coordinates calculated as follows:

$$U = k_0(\phi - \phi_m) \quad \text{and} \quad V = k_0(\lambda - \lambda_m)$$

Where ϕ_m and λ_m are the coordinates of the approximate centre of the region. The parameters A_{ij} , B_{ij} , k_0 , ϕ_m and λ_m are given in table 2 below. The transformed geographical coordinates are then obtained as follows:

$$\lambda_{\text{ETRS}} = \lambda_{\text{IG}} + \Delta\lambda \quad \text{and} \quad \phi_{\text{ETRS}} = \phi_{\text{IG}} + \Delta\phi$$

The reverse transformation from ETRS89 to Irish Grid cannot be calculated directly and requires iteration.

Conversions between geographical and grid coordinates are computed using standard Transverse Mercator projection formulae in association with the published Irish Grid parameters.

Table of coefficients for OSi/OSNI polynomial transformation

Coefficient i, j	Latitude (Ai, j)	Longitude (Bi, j)
0, 0	0.763	-2.810
0, 1	0.123	-4.680
0, 2	0.183	0.170
0, 3	-0.374	2.163
1, 0	-4.487	-0.341
1, 1	-0.515	-0.119
1, 2	0.414	3.913
1, 3	13.110	18.867
2, 0	0.215	1.196
2, 1	-0.570	4.877
2, 2	5.703	-27.795
2, 3	113.743	-284.294
3, 0	-0.265	-0.887
3, 1	2.852	-46.666
3, 2	-61.678	-95.377
3, 3	-265.898	-853.950

Other parameters

$$\phi_m = 53.5$$

$$\lambda_m = -7.7$$

$$K_0 = 0.1$$

The Ordnance Survey Geoid model: OSGM02

Orthometric height (h) in the UK and Ireland can be found by the formula:

$$h = H - N$$

Where H is the GRS80 ellipsoidal height, and N is the geoid undulation (geoid-ellipsoid separation).

Please note: some publications use the notations of h and H the other way round.

OSGM02 uses a grid look-up method, with geoid undulation interpolated from a 1 km grid covering the UK and the Republic of Ireland.

Similar to the Ordnance Survey (Great Britain) OSTN02 transformation, the first stage in calculating the geoid undulation is to identify in which grid cell the ETRS89 point lies. To identify the appropriate grid cell and record numbers in the GB data set follow the procedure given earlier in 'Transforming ETRS89 coordinates to OSGB36 National Grid'.

To identify the appropriate grid cell and record numbers in the Northern Ireland dataset, use the following formulae:

$$\begin{aligned}\text{east_index} &= (\text{integer_part_of } (x/1000)) - 550 \\ \text{north_index} &= (\text{integer_part_of } (y/1000)) - 800 \\ \text{record_number} &= \text{east_index} + (\text{north_index} \times 251) + 1\end{aligned}$$

To identify the appropriate grid cell and record numbers in the Republic of Ireland dataset, use:

$$\begin{aligned}\text{east_index} &= (\text{integer_part_of } (x/1000)) - 400 \\ \text{north_index} &= (\text{integer_part_of } (y/1000)) - 500 \\ \text{record_number} &= \text{east_index} + (\text{north_index} \times 351) + 1\end{aligned}$$

For both Irish datasets the ETRS89 eastings (x) and ETRS89 northings (y) must be computed using the GRS80 ellipsoid and ITM projection (see [appendix B](#)).

Having located the correct cell, find the values of the geoid undulations at the four corners of the cell (sg_0, sg_1, sg_2, sg_3) and the offsets of the point (dx, dy) from the bottom left corner of the cell (x_0, y_0) – as shown in figure 1 above (the se and sn values are used to find the OSTN02 eastings and northings shifts).

The value of the geoid undulation at point x, y is given as follows:

$$N = (1 - t)(1 - u) sg_0 + (t)(1 - u) sg_1 + (t)(u)sg_2 + (1 - t)(u) sg_3$$

Where: $t = dx / 1\ 000$ and $u = dy / 1\ 000$

The resulting geoid undulation is subtracted from the ellipsoidal height (H) to give orthometric height (h).

Chapter 4 Quality statement

Coverage

OSTN02 covers Great Britain and the Isle of Man. The OSi/OSNI polynomial transformation covers the Republic of Ireland and Northern Ireland. It should be noted that the Irish Grid and the National Grid are two independent coordinate reference systems, and that Irish Grid coordinates are not directly compatible with OSGB36 coordinates.

OSGM02 covers all of Great Britain, Isle of Man, Republic of Ireland, and Northern Ireland. The Geoid model comprises 14 patches in order to relate to mean sea level as defined by the specific vertical datum for each region. The datum flag that forms part of each data record specifies to which datum the geoid-ellipsoid separation value relates.

Both models have been *cookie cut* to a boundary that extends 10 km offshore and 2 km beyond the Northern Ireland/Republic of Ireland border. Any point outside this boundary will return null values in the shift and datum flag records. **It is strongly suggested that any software written to incorporate this data be capable of recognising a null value and to return an *outside of model boundary* error message.**

Within Ireland and Northern Ireland, OSGM02 returns orthometric heights relative to the Malin Head and Belfast Lough datums respectively. OSGM02 will return orthometric height relative to either the Malin Head or the Belfast Lough datums for points within 2 km of the border between the Republic of Ireland and Northern Ireland.

Accuracy of Ordnance Survey transformations

Within Great Britain, OSTN02 is the definitive OSGB36/ETRS89 transformation. **OSTN02 in combination with the ETRS89 coordinates of the active GPS network stations, rather than the fixed triangulation network, now define the National Grid.** This means that, for example, the National Grid coordinates of an existing OSGB36 point, refixed using GPS from the National GPS Network and OSTN02, will be the correct ones. The original archived OSGB36 National Grid coordinates of the point (if different) will no longer be true OSGB36, by definition, but the two coordinates (new and archived) will agree on average to better than 0.1 m, (68% probability).

Within the Republic of Ireland and Northern Ireland the OSi/OSNI polynomial transformation is recommended for coordinate transformations between Irish Grid and ETRS89. Transformed ETRS89 coordinates will agree with Irish Grid coordinates derived from traditional survey control to within 0.4 m (95% data).

Accuracy of OSGM02

The heights output by precise GPS positioning in the ETRS89 coordinate system are geometric distance above the WGS84 (GRS80) reference ellipsoid. Note that GPS heights are typically two to three times less precise than horizontal positions. OSGM02 converts GPS ellipsoid heights to orthometric heights above mean sea level.

In mainland Great Britain, the datum (origin point) representing mean sea level is Ordnance Datum Newlyn, defined at Newlyn in Cornwall. In the Republic of Ireland, Northern Ireland, and the islands surrounding Great Britain, mean sea level is defined by specific independent vertical datums that are all incorporated in OSGM02 and hence OSGM02 is compatible with the products from each of the Ordnance Surveys. Other geoid models may give mean sea level heights that are incompatible with the Ordnance Surveys' products.

The estimated accuracies of OSGM02 for each regional vertical datum are included in the table below. The figures quoted assume precise ellipsoidal heights are used; for lower quality GPS observations additional error budget must be included.

Regional datum	Standard error (m)
Great Britain	0.02
Republic of Ireland	0.03
Northern Ireland	0.02
Orkney	0.08
Shetland	0.03
Outer Hebrides	0.09
Isle of Man	0.03
St Kilda	0.06

Any discrepancy found between an Ordnance Survey levelled bench mark and a OSGM02 computed orthometric height is likely to be due to bench mark subsidence or uplift and, assuming precise GPS survey has been carefully carried out, **the orthometric height given by OSGM02 should be considered correct in preference to archive bench mark heights.**

Appendix A Transforming ETRS89 GPS coordinates to OSGB36 and orthometric height

Worked example

To convert the coordinates of Caister Water Tower (at position given by ETRS89 geographical coordinates 52° 39' 28.8282" N, 1° 42' 57.8663" E, 108.05 m) to OSGB36 and orthometric height.

Step 1: Compute ETRS89 eastings and northings – see [appendix B](#)

Latitude = 52° 39' 28.8282" N
= 52.658007833° N

Longitude = 1° 42' 57.8663" E
= 1.716073973° E

The parameters for the GRS80 ellipsoid are:

a = 6 378 137.000
b = 6 356 752.314

Following the procedure in [appendix B](#), the calculation steps yield the following:

e^2 = 6.69437999e-03
 ν = 6.38912542e+06
 ρ = 6.37332179e+06
 η^2 = 2.47965408e-03
M = 4.06772557e+05
P = 6.48577261e-02
I = 3.06772557e+05
II = 1.54055171e+06
III = 1.56081387e+05
IIIA = -2.06739447e+04
IV = 3.87545974e+06
V = -1.70023086e+05
VI = -1.01356325e+05
eastings = 651 307.0031
northings = 313 255.6859

The ETRS89 eastings and northings (to the nearest mm) are therefore:

$$\begin{aligned}x &= 651\,307.003 \\y &= 313\,255.686\end{aligned}$$

Step 2: Convert ETRS89 eastings and northings to OSGB36 and ETRS89 height to orthometric height

First calculate the grid cell in which the point lies:

$$\begin{aligned}\text{east_index} &= \text{integer_part_of}(x/1\,000) \\&= 651\end{aligned}$$

$$\begin{aligned}\text{north_index} &= \text{integer_part_of}(y/1\,000) \\&= 313\end{aligned}$$

The eastings and northings of the south-west corner of the cell are therefore:

$$x_0, y_0 = (651\,000, 313\,000)$$

The easting, northing and geoid shifts for the four corners of the cell are given by:

$$\begin{aligned}(se_0, sn_0, sg_0) &= \text{shifts}(\text{east_index}, \text{north_index}) \\&= \text{shifts}(651, 313) \\&= \text{record}(651 + (313 \times 701) + 1) \\&= \text{record}(220\,065) \\&= (102.775, -78.244, 44.252)\end{aligned}$$

$$\begin{aligned}(se_1, sn_1, sg_1) &= \text{shifts}(652, 313) \\&= \text{record}(220\,066) \\&= (102.813, -78.246, 44.236)\end{aligned}$$

$$\begin{aligned}(se_2, sn_2, sg_2) &= \text{shifts}(652, 314) \\&= \text{record}(220\,767) \\&= (102.822, -78.227, 44.224)\end{aligned}$$

$$\begin{aligned}(se_3, sn_3, sg_3) &= \text{shifts}(651, 314) \\&= \text{record}(220\,766) \\&= (102.783, -78.216, 44.240)\end{aligned}$$

The offset values are given by:

$$\begin{aligned} dx &= x - x_0 \\ &= 307.0032 \end{aligned}$$

$$\begin{aligned} dy &= y - y_0 \\ &= 255.6860 \end{aligned}$$

$$\begin{aligned} t &= dx/1\,000 \\ &= 0.3070032 \end{aligned}$$

$$\begin{aligned} u &= dy/1\,000 \\ &= 0.2556860 \end{aligned}$$

The shifts are therefore:

$$\begin{aligned} se &= (1-t)(1-u) se_0 + (t)(1-u) se_1 + (t)(u) se_2 + (1-t)(u) se_3 \\ &= 102.789 \end{aligned}$$

$$\begin{aligned} sn &= (1-t)(1-u) sn_0 + (t)(1-u) sn_1 + (t)(u) sn_2 + (1-t)(u) sn_3 \\ &= -78.238 \end{aligned}$$

$$\begin{aligned} sg &= (1-t)(1-u) sg_0 + (t)(1-u) sg_1 + (t)(u) sg_2 + (1-t)(u) sg_3 \\ &= 44.244 \end{aligned}$$

And finally, the National Grid (OSGB36) eastings and northings coordinates are given by:

$$\begin{aligned} e &= x + se \\ &= 651\,409.792 \end{aligned}$$

$$\begin{aligned} n &= y + sn \\ &= 313\,177.448 \end{aligned}$$

The orthometric height h is given by

$$\begin{aligned} h &= 108.05 - sg \\ &= 108.05 - 44.244 \\ &= 63.806 \end{aligned}$$

So Caister Water Tower has National Grid (OSGB36) coordinates **(651 409.792, 313 177.448)** and orthometric height 63.806 m relative to the vertical datum as indicated by the datum flag field – which in this case = 1, indicating Ordnance Survey Datum Newlyn.

Using the procedure in [appendix C](#), these coordinates can be converted to latitude and longitude. A worked example of this step is given in [appendix C](#).

Inverse transformation: OSGB36 to ETRS89

Worked example

Taking the OSGB36 coordinates from the example above, that is **651 409.792, 313 177.448**, the procedure for the inverse transformation (OSGB36 to ETRS89) gives the following iterative solution:

First values of OSGB36 to ETRS89 shifts: -102.792064 east, 78.240505 north.

First values of ETRS89 easting and northing: 651 306.999936 east, 313 255.688505 north.

Second values of OSGB36 to ETRS89 shifts: -102.788790 east, 78.238161 north.

Second values of ETRS89 easting and northing: 651 307.003210 east, 313 255.686161 north.

Third values of OSGB36 to ETRS89 shifts: -102.788790 east, 78.238161 north.

Third values of ETRS89 easting and northing: 651 307.003210 east, 313 255.686161 north.

Since the second and third iterations show convergence at the required level, the calculation is stopped.

Feeding the ETRS89 coordinates **(651 307.003210, 313 255.686161)** back into the procedure for transforming coordinates gives the OSGB36 coordinates **(651 409.792000, 313 177.448000)**, showing that, for this example, applying the algorithm in both directions gives the values that were started with.

To calculate the GRS80 ellipsoidal height from the orthometric height simply add the geoid-ellipsoid separation.

Appendix B Converting latitude and longitude to easting and northing

The formulae in this appendix and [appendix C](#) require ellipsoid constants and projection constants, given in the tables below.

Important note: When converting OSGB36 coordinates between (easting, northing) and (latitude, longitude) in either direction, use the Airy 1830 ellipsoid constants. When converting ETRS89 coordinates between (easting, northing) and (latitude, longitude) in either direction, use the GRS80 ellipsoid constants. Use the same National Grid projection constants for both ETRS89 and OSGB36 coordinates.

The ITM (Irish Transverse Mercator) projection is required to obtain ETRS89 eastings and ETRS89 northings for use with the OSGM02 Geoid model data files for Northern Ireland and the Republic of Ireland. The ITM projection should only be used with the GRS80 ellipsoid.

Ellipsoid constants

Ellipsoid	Semi-major axis a (metres)	Semi-minor axis b (metres)	Used for the following coordinate system
Airy 1830	6 377 563.396	6 356 256.910	OSGB36 National Grid
GRS80‡	6 378 137.000	6 356 752.3141	ETRS89 (WGS84)

‡ Also known as the WGS84 ellipsoid.

The ellipsoid squared eccentricity constant e^2 is computed from a and b by:

$$e^2 = \frac{a^2 - b^2}{a^2} \quad (\text{B1})$$

Projection constants

Projection	Scale factor on central meridian (F_0)	True origin (ϕ_0 and λ_0)	Map coordinates of true origin (m) (E_0 and N_0)
National Grid	0.9996012717	lat 49° N long 2° W	E 400 000 N 100 000
ITM	0.99982	Lat 53° 30' N long 8° W	E 600 000 N 750 000

To convert a position from the graticule of latitude and longitude coordinates (λ, ϕ) to a grid of easting and northing coordinates (E, N) using a transverse mercator projection, for example OSGB36 National Grid, ITM or UTM (Universal Transverse Mercator), compute the following formulae. Remember to express all angles in radians. You will need the ellipsoid constants a , b and e^2 and the projection constants listed below:

N_0 – northing of true origin;

E_0 – easting of true origin;

F_0 – scale factor on central meridian;

ϕ_0 – latitude of true origin; and

λ_0 – longitude of true origin and central meridian.

$$n = \frac{a-b}{a+b} \quad (\text{B2})$$

$$v = aF_0(1 - e^2 \sin^2 \phi)^{-0.5} \quad (\text{B3})$$

$$\rho = aF_0(1 - e^2)(1 - e^2 \sin^2 \phi)^{-1.5} \quad (\text{B4})$$

$$\eta^2 = \frac{v}{\rho} - 1 \quad (\text{B5})$$

$$M = bF_0 \left(\begin{aligned} &\left(1 + n + \frac{5}{4}n^2 + \frac{5}{4}n^3\right)(\phi - \phi_0) - \left(3n + 3n^2 + \frac{21}{8}n^3\right)\sin(\phi - \phi_0)\cos(\phi + \phi_0) \\ &+ \left(\frac{15}{8}n^2 + \frac{15}{8}n^3\right)\sin(2(\phi - \phi_0))\cos(2(\phi + \phi_0)) - \frac{35}{24}n^3\sin(3(\phi - \phi_0))\cos(3(\phi + \phi_0)) \end{aligned} \right) \quad (\text{B6})$$

$$I = M + N_0$$

$$II = \frac{\nu}{2} \sin \phi \cos \phi$$

$$III = \frac{\nu}{24} \sin \phi \cos^3 \phi (5 - \tan^2 \phi + 9\eta^2)$$

$$IIIA = \frac{\nu}{720} \sin \phi \cos^5 \phi (61 - 58 \tan^2 \phi + \tan^4 \phi)$$

$$IV = \nu \cos \phi$$

$$V = \frac{\nu}{6} \cos^3 \phi \left(\frac{\nu}{\rho} - \tan^2 \phi \right)$$

$$VI = \frac{\nu}{120} \cos^5 \phi (5 - 18 \tan^2 \phi + \tan^4 \phi + 14\eta^2 - 58 \tan^2 \phi \eta^2)$$

$$N = I + II(\lambda - \lambda_0)^2 + III(\lambda - \lambda_0)^4 + IIIA(\lambda - \lambda_0)^6 \quad (\text{B7})$$

$$E = E_0 + IV(\lambda - \lambda_0) + V(\lambda - \lambda_0)^3 + VI(\lambda - \lambda_0)^5 \quad (\text{B8})$$

Worked example using the Airy 1830 ellipsoid and National Grid

Intermediate values are shown here to 10 decimal places. Compute all values using double-precision arithmetic.

ϕ	52° 39' 27.2531" N
λ	1° 43' 4.5177" E
ν	6.3885023333e+06
ρ	6.3727564399e+06
η^2	2.4708136169e-03
M	4.0668829596e+05
I	3.0668829596e+05
II	1.5404079092e+06
III	1.5606875424e+05
IIIA	-2.0671123011e+04
IV	3.8751205749e+06
V	-1.7000078208e+05
VI	-1.0134470432e+05
E	651 409.903 m
N	313 177.270 m

Appendix C Converting easting and northing to latitude and longitude

Obtaining (λ, ϕ) from (E, N) is an iterative procedure. You need values for the ellipsoid and projection constants $a, b, e^2, N_0, E_0, F_0, \phi_0$ and λ_0 given in [appendix B](#). Remember to express all angles in radians.

First compute:

$$\phi' = \left(\frac{N - N_0}{aF_0} \right) + \phi_0 \quad (\text{C1})$$

and M from equation (B6) in [appendix B](#), substituting ϕ' for ϕ .

If the absolute value of $(N - N_0 - M) \geq 0.01$ mm, obtain a new value for ϕ' using:

$$\phi'_{\text{new}} = \left(\frac{N - N_0 - M}{aF_0} \right) + \phi' \quad (\text{C2})$$

and recompute M substituting ϕ' for ϕ .

Iterate until the absolute value of $(N - N_0 - M) < 0.01$ mm, then compute ρ , ν and η^2 using equations (B3, B4 and B5) in [appendix B](#) and compute:

$$\text{VII} = \frac{\tan \phi'}{2\rho\nu}$$

$$\text{VIII} = \frac{\tan \phi'}{24\rho\nu^3} (5 + 3 \tan^2 \phi' + \eta^2 - 9 \tan^2 \phi' \eta^2)$$

$$IX = \frac{\tan \phi'}{720 \rho v^5} (61 + 90 \tan^2 \phi' + 45 \tan^4 \phi')$$

$$X = \frac{\sec \phi'}{v}$$

$$XI = \frac{\sec \phi'}{6v^3} \left(\frac{v}{p} + 2 \tan^2 \phi' \right)$$

$$XII = \frac{\sec \phi'}{120v^5} (5 + 28 \tan^2 \phi' + 24 \tan^4 \phi')$$

$$XIIA = \frac{\sec \phi'}{5040v^7} (61 + 662 \tan^2 \phi' + 1320 \tan^4 \phi' + 720 \tan^6 \phi')$$

$$\phi = \phi' - VII(E - E_0)^2 + VIII(E - E_0)^4 - IX(E - E_0)^6 \quad (C3)$$

$$\lambda = \lambda_0 + X(E - E_0) - XI(E - E_0)^3 + XII(E - E_0)^5 - XIIA(E - E_0)^7 \quad (C4)$$

Worked example using Airy 1830 ellipsoid and National Grid

Intermediate values are shown here to 10 decimal places. Compute all values using double precision arithmetic.

E	651 409.903 m
N	313 177.270 m
ϕ'	9.2002324604e-01 rad
M	4.1290347144e+05
2nd ϕ'	9.2006619470e-01 rad
M	4.1317717541e+05
3rd ϕ'	9.2006620953e-01 rad
M	4.1317726997e+05
final ϕ'	9.2006620954e-01 rad
ν	6.3885233415e+06 m
ρ	6.3728193094e+06 m
η^2	2.4642205195e-03
VII	1.6130562490e-14
VIII	3.3395547442e-28
IX	9.4198561719e-42
X	2.5840062509e-07
XI	4.6985969968e-21
XII	1.6124316622e-34
XIIA	6.6577316330e-48
ϕ	52° 39' 27.2531" N
λ	1° 43' 4.5177" E

Appendix D Glossary

The following is a list of technical terms used in this user guide, together with a fuller definition.

datum

A point, line, surface or set of these, with respect to which positions of objects can be stated as unique sets of coordinates.

de facto national standard

A national standard by adoption rather than legally enforced.

ellipsoid (biaxial)

The 3-D geometric figure obtained by rotating an ellipse about its minor axis. Used in geodesy to approximate the shape of the earth.

ETRF89

European Terrestrial Reference Frame 1989 – the Europe-fixed realisation of WGS84. Governed by EUREF as a standard reference frame for Europe.

ETRS89

European Terrestrial Reference System 1989 – a coordinate system that is the Europe-fixed precise version of WGS84. Governed by EUREF as the standard fixed reference system for Europe. ETRS89 is related to the state-of-the-art WGS84-consistent system ITRS2000 by a six-parameter kinematic transformation published by IERS.

EUREF

EUREF (European Reference Frame): a sub-commission of the International Association of Geodesy, Commission X.

geocentric datum

A reference system that uses the centre of mass of the earth as its origin; the popularity of these systems today derives from their usefulness in describing satellite orbits.

Geoid model

A model of the level surface which is closest to mean sea level over the oceans. This surface is continued under land as the fundamental reference surface for height measurement.

GPS

Global Positioning System – an outdoor positioning technique using a constellation of US Department of Defense satellites and a portable receiver to dynamically determine coordinates. For high precision, several receivers are used and their relative positions are determined.

GRS80

A global reference ellipsoid used in the WGS84 coordinate system. Also known as the WGS84 ellipsoid.

IERS

International Earth Rotation Service.

ITRF

International Terrestrial Reference Frame – the state-of-the-art global realisation of the WGS84 reference system, using observations from worldwide networks of active geodetic stations of the VLBI, SLR, GPS and DORIS techniques.

ODN

Ordnance Datum Newlyn – the levelling-based vertical reference frame for most of the British Isles, with a single tide gauge constraint in Newlyn in Cornwall.

OSGB36

Ordnance Survey Great Britain 1936 – the British horizontal mapping datum, observed by triangulation from 1936 and traditionally realised on the ground by triangulation stations. With the release of the definitive transformation, OSTN02, OSGB36 is now realised by the ETRS89 coordinates of the National GPS Network in conjunction with the OSTN02 transformation.

OSGM02

Ordnance Survey National Geoid Model 2002 – a gravimetric Geoid model that is aligned with the national height datums of Great Britain, Northern Ireland and Ireland.

OSTN02

Ordnance Survey National Grid Transformation 2002 – a grid shift type horizontal transformation between the ETRS89 datum and OSGB36 National Grid.

realisation

A spatial reference system made real on the ground by monumented points with estimated coordinates and errors.

transformation

A procedure to change from one coordinate system to another.

WGS84

World Geodetic System 1984 – the global geodetic reference system used to describe the position of GPS satellites and ground stations.